Characterizing AS Relationships by Recursive Analysis of Measured AS Adjacency Matrix

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Summary

- Quantify AS magnitude
  - quantify AS' network scale by using traffic transition model based on degree
    - To calculate the magnitude, we use eigenvalue analysis.
- Characterize AS relationships
  - analyze difference of magnitude by AS relationships
    - towards path-less annotation
    - show potential of finding inaccurate annotations in CAIDA's algorithm

AS relationships

- two major types
  - transit
    - provider-customer relationship
      - provider AS : larger network
      - customer AS : smaller network
  - peering
    - peer-to-peer relationship
      - among equal-scale networks

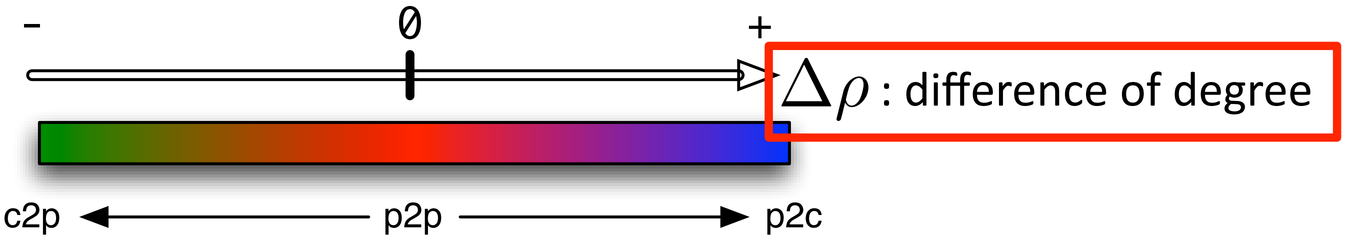
Related work

- AS relationships inference based on "*valley-free path model*"
  - heuristics [Gao 2001]
    - annotate links, eliminating contradiction to valley-free path model by analyzing AS paths in routing tables
  - (weighted) MAX2SAT [Battista et al. 2003, 2007, Dimitropoulos et al. 2005, 2007]
    - maximize the (weighted) number of valley-free paths in routing tables

These researches classify the relationships into two or three (+sibling) types.
→ Characterize the relationships **quantitatively**

Well-known way to represent the relationships quantitatively

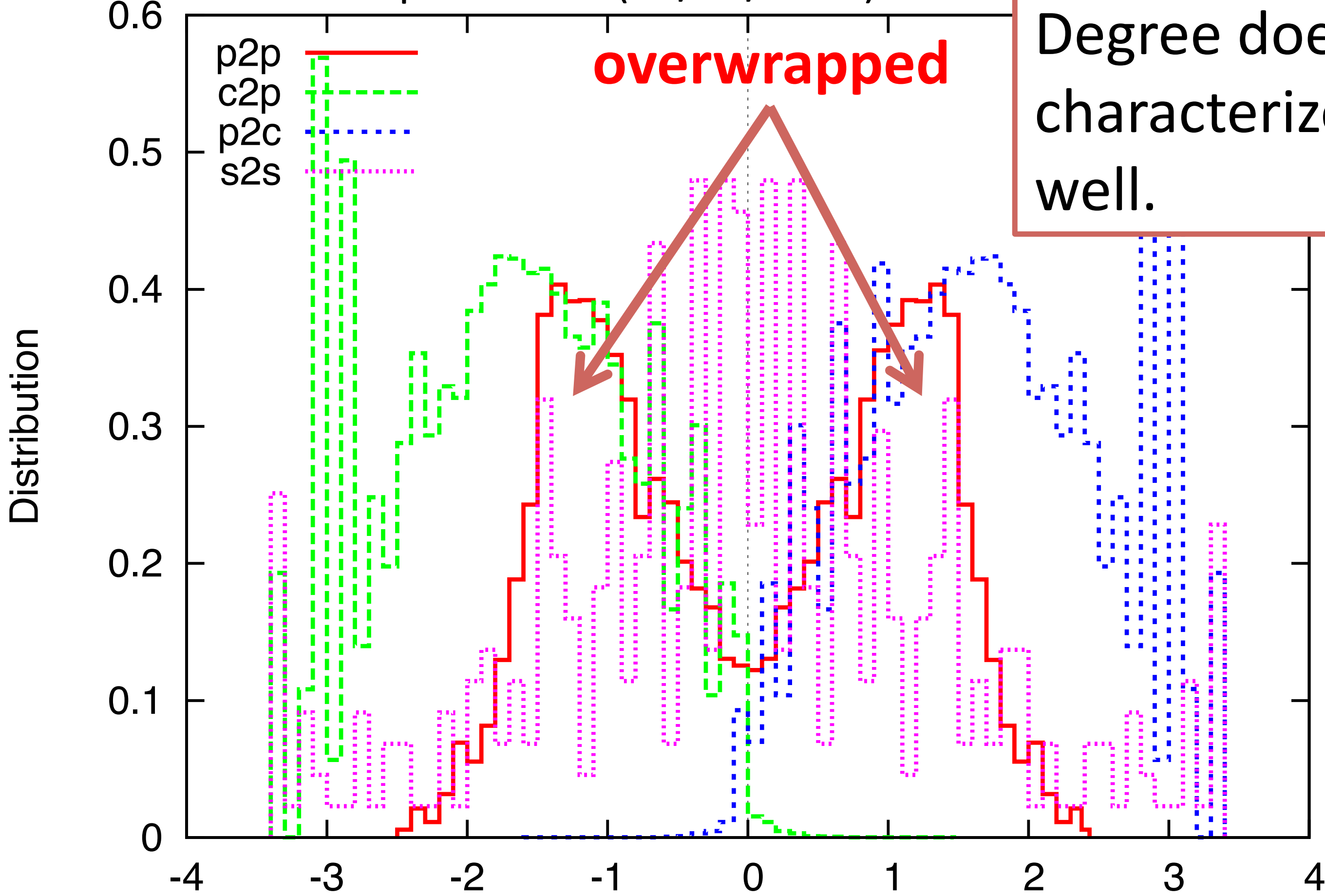
- Degree; i.e., #number of neighbors
  - high degree = larger AS
    - Larger AS tends to be provider.
  - low degree = smaller AS
    - Smaller AS tends to be customer.



p2c: provider to customer
c2p: customer to provider
p2p: peer to peer

In reality…

Dataset: CAIDA AS Relationships Dataset (10/08/2009)



Degree does not characterize peering well.

$\Delta\rho$: difference of degree in log-arithmetic

Note; the distribution is normalized by area for each type of relationships.

AS Magnitude Quantification

- AS magnitude
  - representing network scale
    - e.g., degree (but more appropriate)
- concept of AS magnitude quantification
  - take into account the scale of neighbor ASes
    - e.g., An AS connecting to larger ASes is also larger, even though the AS has low degree.

How do we calculate the magnitude?

- employ traffic transition model based on "degree"
  - What does degree mean? (an aspect)
    - The degree vector multiplied by a constant is calculated as the left eigenvector of AS adjacency (stochastic) matrix corresponding to the maximum eigenvalue.
      - i.e., degree = traffic distribution in random walk model

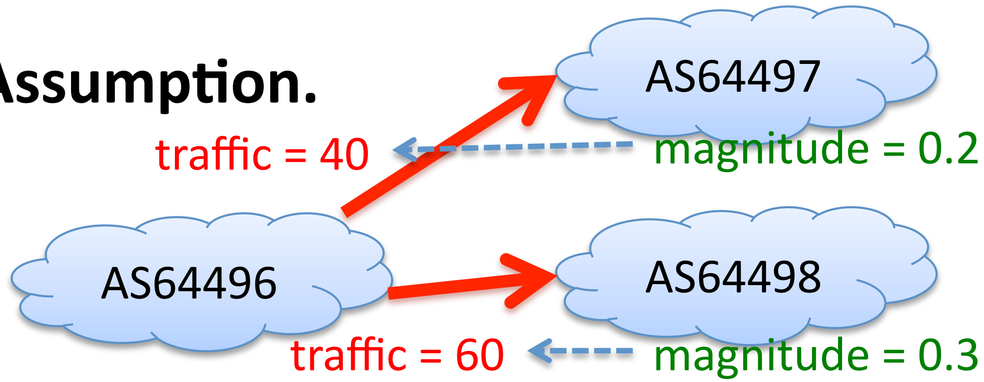
→ extend this into non-random walk model

Extension from random walk to non-random walk

- non-random walk model
  - egress traffic (probability of transitioning to a neighbor)
    - proportional to the neighbor AS's magnitude
      - in random walk, the probability is uniform at every neighbors

Note; the magnitude is defined recursively

Assumption.



AS magnitude quantification - calculation procedure

Idea: calculate the traffic distribution and map it to the magnitude

(1) Define a weighted AS adjacency matrix

$$ {}^{n}A := \begin{pmatrix} {}^{n}a_{11} & \cdots & {}^{n}a_{1j} & \cdots & {}^{n}a_{1m} \\ \vdots & \ddots & \vdots & & \vdots \\ {}^{n}a_{i1} & \cdots & {}^{n}a_{ij} & \cdots & {}^{n}a_{im} \\ \vdots & & \vdots & \ddots & \vdots \\ {}^{n}a_{m1} & \cdots & {}^{n}a_{mj} & \cdots & {}^{n}a_{mm} \end{pmatrix} $$

(i) $n = 0$ **random walk model for initial case**

$$ {}^{n}a_{ij} = \begin{cases} 1 & : \text{if AS } i \text{ and AS } j \text{ are adjacent} \\ 0 & : \text{otherwise} \end{cases} $$

(ii) $n \geq 1, n \in \mathbb{Z}$

$$ {}^{n}a_{ij} = \begin{cases} {}^{(n-1)}\rho_j & : \text{if AS } i \text{ and AS } j \text{ are adjacent} \\ 0 & : \text{otherwise} \end{cases} $$

recursive definition

(2) Equalize ingress and egress traffic; i.e., converting to **traffic transition matrix**

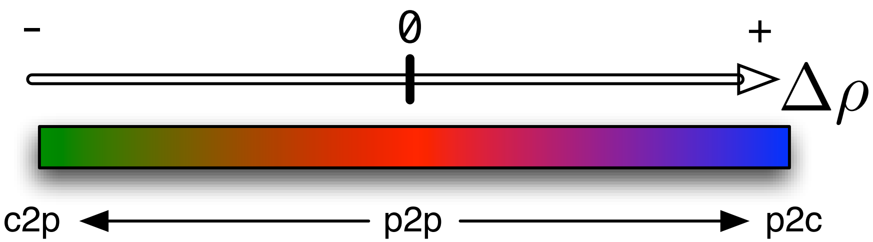
$$ {}^{n}T = \left(\frac{{}^{n}a_{ij}}{\sum_k {}^{n}a_{ik}} \right) $$

(3) Calculate the left eigenvector of *T* corresponding to the maximum eigenvalue

$ {}^{n}\boldsymbol{\rho} $: the left eigenvector; the *i*-th element denotes the magnitude of AS *i*.

AS relationships estimation: the difference of magnitude

Idea: estimate the relationships from difference of magnitude



p2c: provider to customer
c2p: customer to provider
p2p: peer to peer

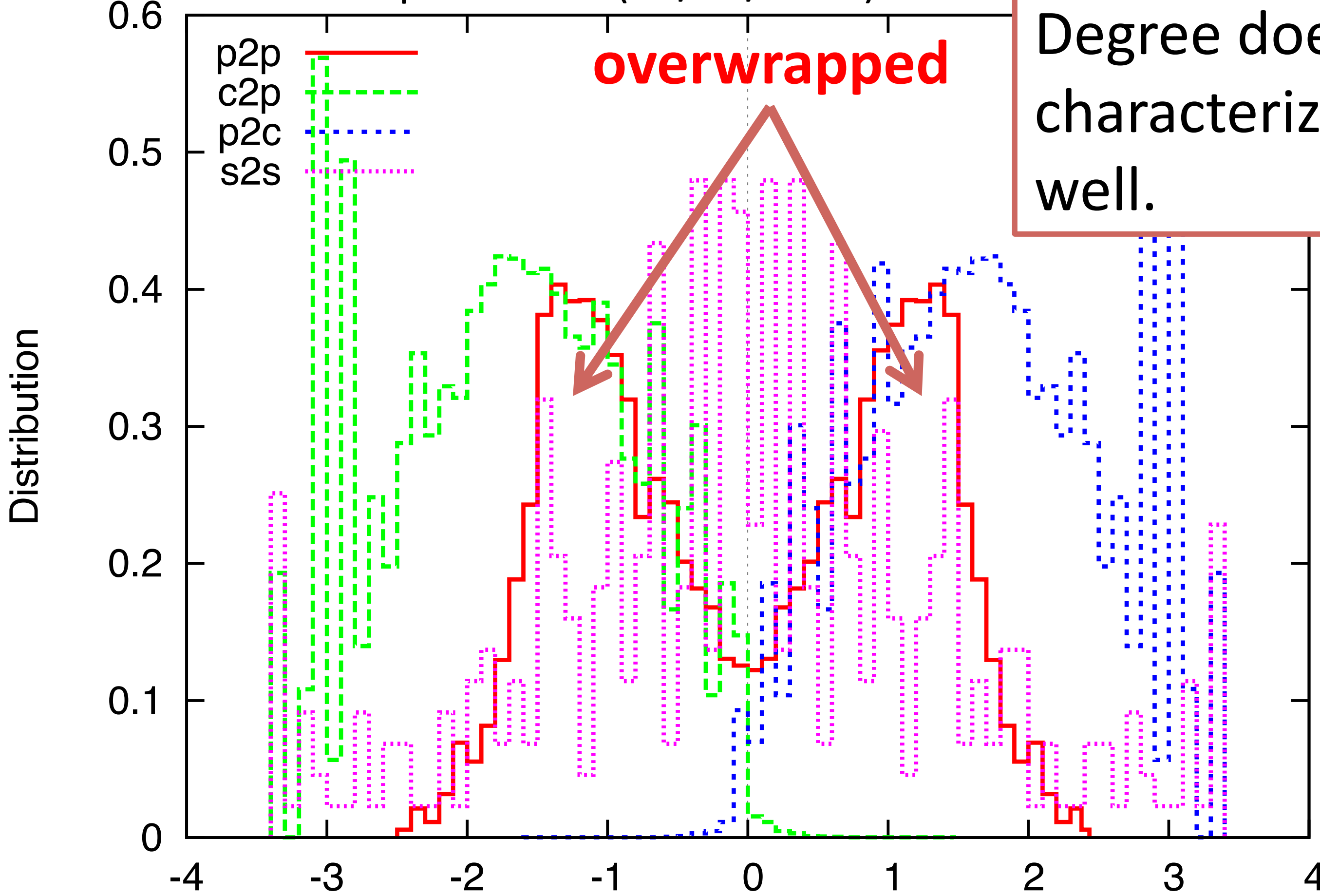
$$\Delta^{n}\rho_{i,j} := \log_{10}\left(\frac{{}^{n}\rho_i}{{}^{n}\rho_j}\right)$$

$$= \log_{10}\left({}^{n}\rho_i\right) - \log_{10}\left({}^{n}\rho_j\right)$$

Analysis & Evaluation

Back to degree-based representation

Dataset: CAIDA AS Relationships Dataset (10/08/2009)



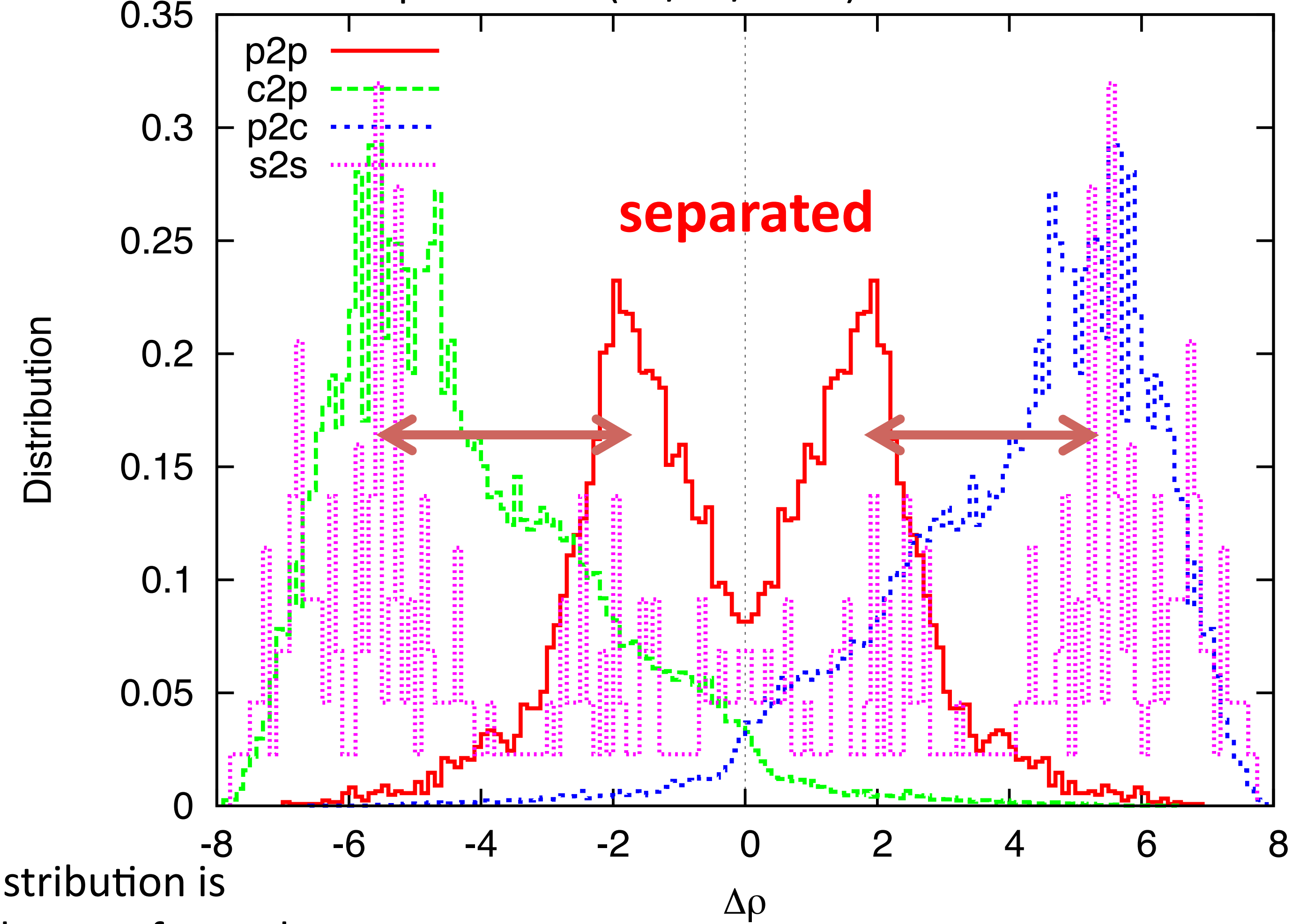
Degree does not characterize peering well.

Δρ : difference of degree in log-arithmetic

Note; the distribution is normalized by area for each type of relationships.

The distribution of difference of magnitude (n=2)

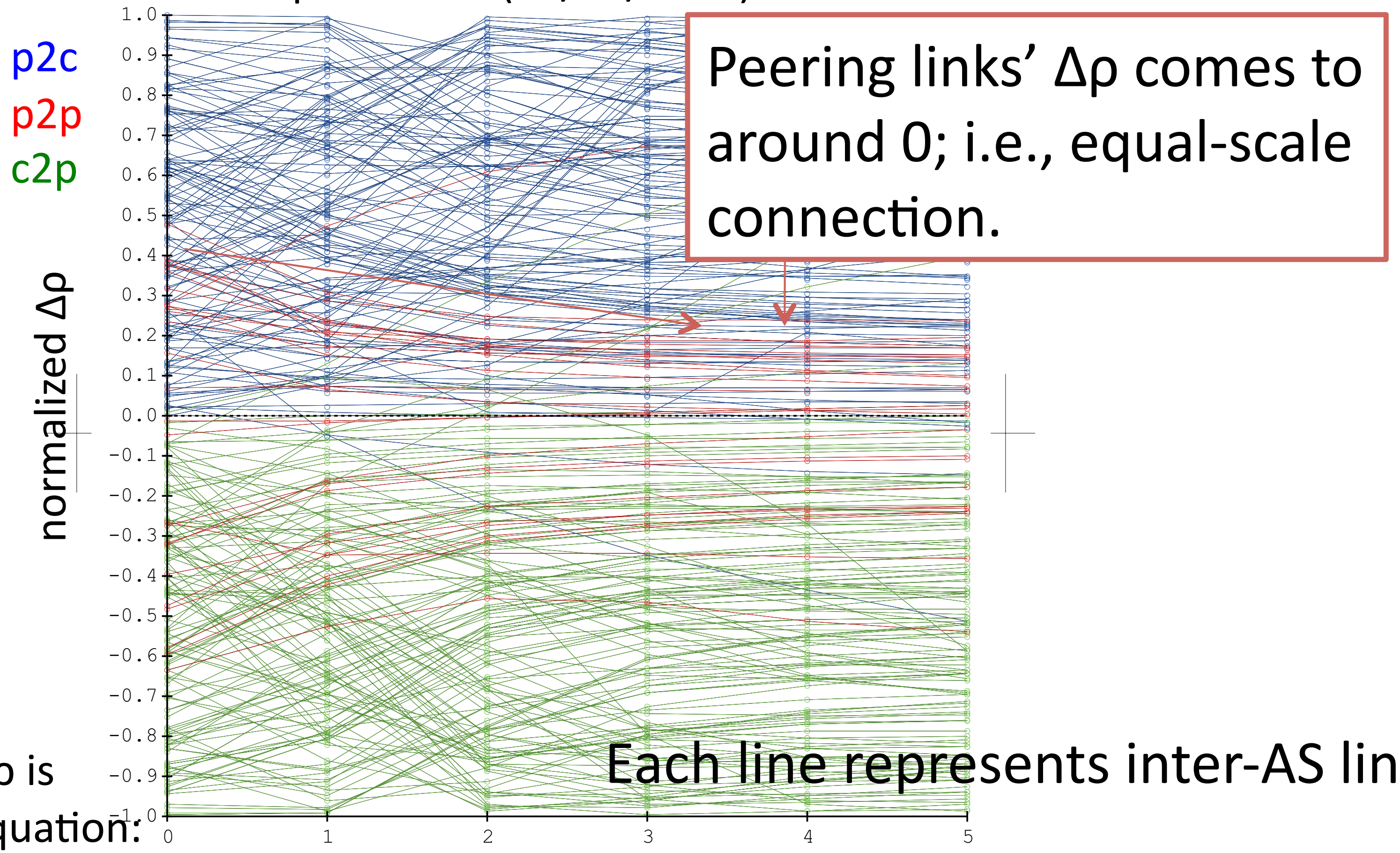
Dataset: CAIDA AS Relationships Dataset (10/08/2009)



Note; the distribution is normalized by area for each type of relationships.

How the difference of magnitude goes?

Dataset: CAIDA AS Relationships Dataset (10/08/2009)



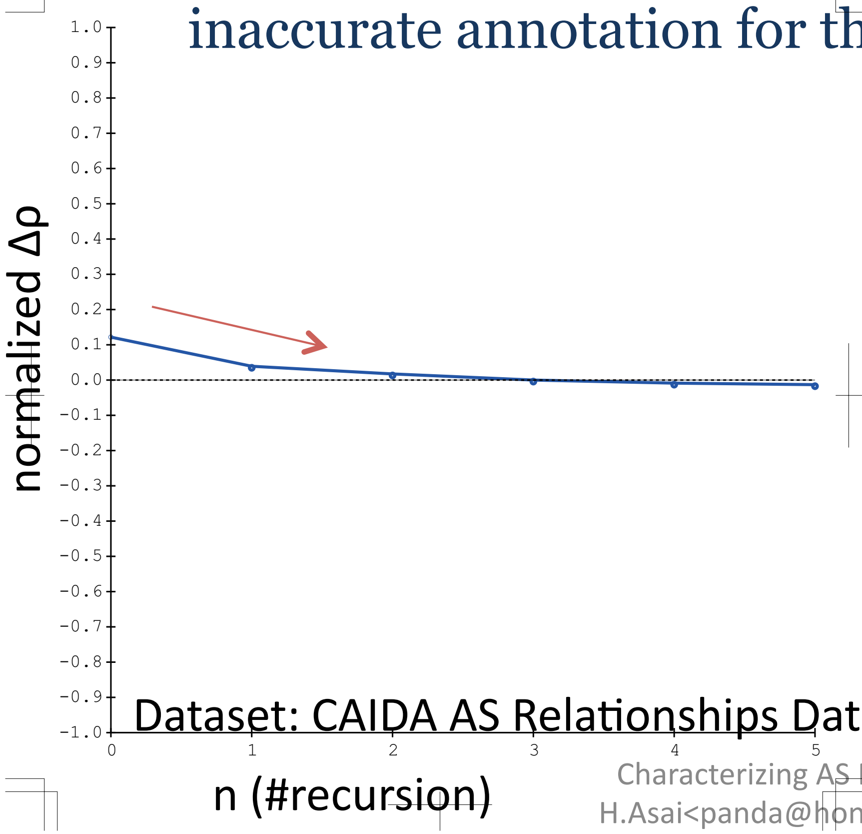
n (#recursion)

(*) randomly sampled at rate 1/500

Note; normalized Δρ is computed by the equation:
rank-of-link/#links – 0.5,
i.e., distributed to the interval [-1:1] uniformly

Potential of finding inaccurate annotations: Is Verison-Verio transit?

- According to CAIDA dataset, Verison (AS701) is provider of Verio (AS2914).
  - Both are considered “Tier-1” ISP. CAIDA’s algorithm made inaccurate annotation for this link?



Dataset: CAIDA AS Relationships Dataset (10/08/2009)

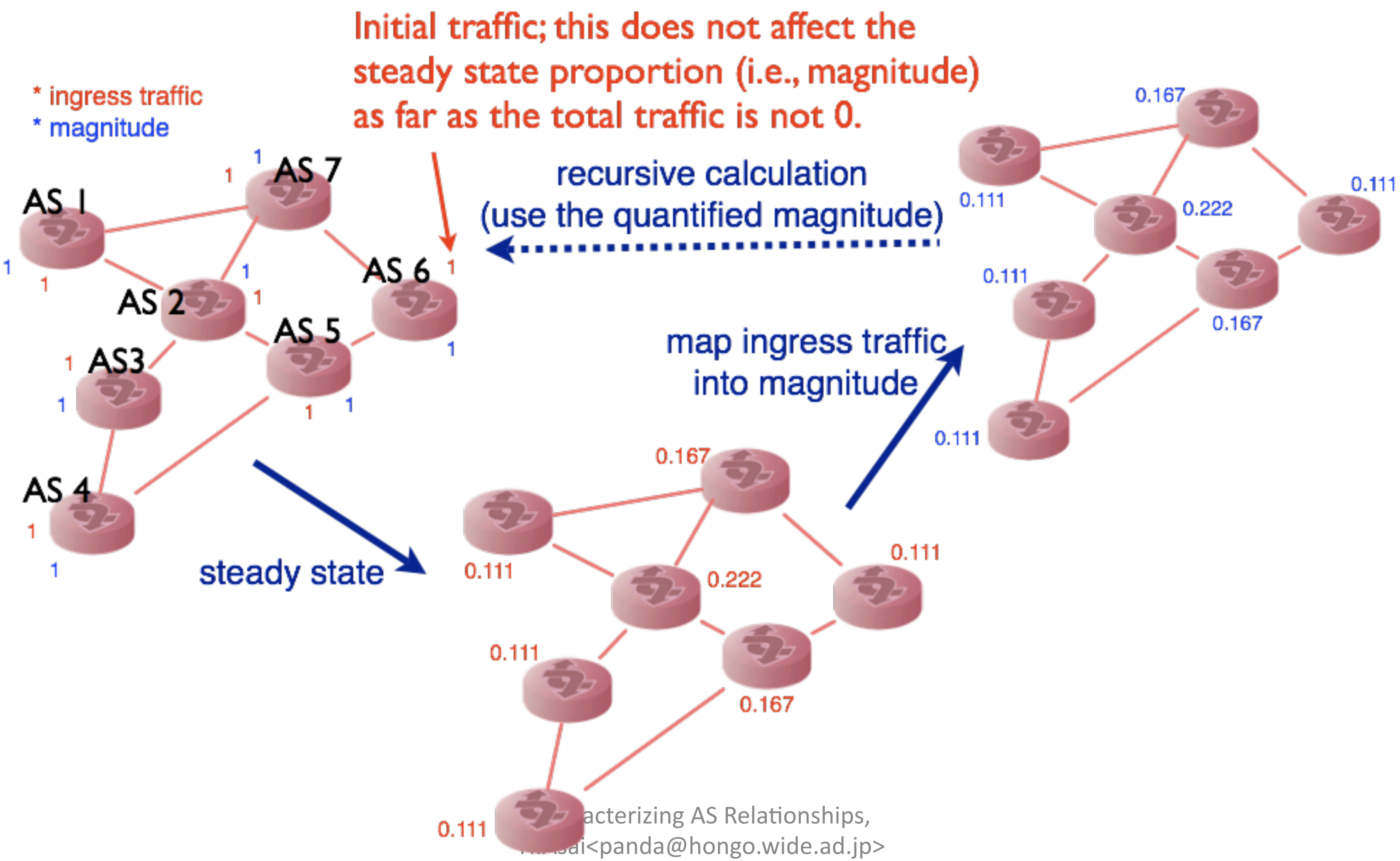
The difference of degree is larger, but the difference of magnitude (n≥1) is smaller.

CAIDA’s annotation may inaccurate. i.e., this link may be peering.

Conclusion

- We presented followings
  - quantify AS magnitude
    - extend degree to magnitude by eigenvalue analysis
  - characterize AS relationships
    - by comparing the difference of magnitude
- contribution
  - proposed path-less (i.e., not path but adjacency) analysis and characterization method for AS relationships
  - showed the potential of finding inaccurate annotations in CAIDA dataset
    - We will evaluate this point in greater detail in future.

Thank you for your attention



Initial traffic; this does not affect the steady state proportion (i.e., magnitude) as far as the total traffic is not 0.
* ingress traffic
* magnitude
AS 1
AS 7
AS 2
AS 6
AS 5
AS3
AS 4
recursive calculation
(use the quantified magnitude)
map ingress traffic into magnitude
steady state
0.167
0.111
0.222
0.111
0.111
0.167
0.111